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# **The Case for a Discriminatory Pricing Rule in Competitive Electricity Pools**

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## **Abstract**

We present a multi-unit common value auction model with capacity constraints which ensure the participants face a residual market. We show that a discriminatory auction performs better than a uniform one when such constraints are present. We then look at a more explicit model of electricity pools and show that the preferred uniform pricing rule can lead to equilibria that are even worse than the basic model suggests. We show that a discriminatory auction would lead to relatively more competitive prices.

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## 1. Introduction

Since 1990, electricity markets around the world have gone through a radical transformation. The benchmark for change was the England and Wales reform which brought to an end the nationalised system that was set up in 1947. Under this system the Central Electricity Generating Board (CEGB) had a monopoly on the supply and high voltage transmission of electricity, while twelve regional electricity boards were responsible for the distribution and sale of electricity. The regional electricity boards were privatised as they were, and became known as the Regional Electricity Companies (RECs). The supply side, however, was restructured. The generating plants of the CEGB were split between 3 companies, National Power, PowerGen and Nuclear Electric. The high voltage transmission network was separated from generation and put in the hands of the National Grid Company, who were also given the role of central dispatcher.

The rationale behind a vertically integrated supply structure is the need for a centralised system which ensures supply and demand are constantly matched at minimum cost subject to network constraints. This requires a central dispatcher to constantly monitor the system and instruct units when to switch on and off. However, a major drawback of an integrated system is there are no competitive market forces acting on the potentially competitive generating system. At the heart of the England and Wales reform was the innovative way in which a competitive generating system was combined with a centralised dispatch system. This was done by setting up an electricity pool which is a spot market for the sale of electricity. Generators compete to supply electricity by submitting bids for the minimum price at which they are willing to supply from each of their plants. The central dispatcher then constructs the least cost rank order of plants for each period (typically an hour or half hour). All units dispatched are paid the marginal price, the bid price of the marginal unit. Hence an electricity pool can be thought of as a multi-unit auction with a uniform pricing rule. A testimony to the success of the England and Wales reform is that similar systems have been adopted and are under consideration elsewhere. The various systems differ in the way they deal with such things as imbalances, network constraints and incentives for investment. In this paper, we restrict attention to the multi-unit auction aspect of electricity pools, a feature they all have in common.

The England and Wales reform has not been without problems. Perhaps the biggest mistake was that the thermal plants of the CEGB were divided between only two companies, National Power and PowerGen. Their plants accounted for 48% and 30% of the total generation capacity available to the pool. Nuclear Electric took control of the nuclear plants which accounted for 14% of capacity. The hope that this market structure would lead to competition in generation was not realised with pool prices above competitive levels. The main problem was the market power of the two large generators, National Power and PowerGen. Most of the theoretical analysis of the England and Wales pool has focused on the relationship between market power and market concentration. Green and Newbery (1992)<sup>2</sup> and von der Fehr and Harbord (1993)<sup>3</sup> show that splitting the thermal generators of the CEGB between more than two companies would have led to lower pool prices. Both papers also show that the present rules lead to equilibria that are not efficient.

In this paper, we argue that for any given number of firms, the market power can be reduced by using a discriminatory auction where the generators are simply paid the price they bid for units that are dispatched. We begin by comparing the pricing rules in a common value, multi-unit auction with capacity constraints which ensure that firms sometimes face a residual demand irrespective of their bids. We show that both pricing rules result in competitive prices when demand is low. However, in high demand periods, when the firms face some residual market, the uniform auction results in much higher prices. Capacity constraints are an important feature of electricity pools. Intuitively, the constraints give firms facing a residual market complete control over the marginal price which is the price all firms get for units sold under a uniform pricing rule. Under a discriminatory pricing rule the marginal price does not carry the same level of significance as it only determines the payment to the marginal unit.

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<sup>2</sup> Green and Newbery (1992) use the supply function framework of Klemperer and Meyer (1989). Using this theoretical framework they simulate the England and Wales spot market to measure the extent and cost of market power. They demonstrate that if the thermal plants had been split between five companies rather than two, equilibrium pool prices would be much closer to competitive levels and hence the deadweight loss would be significantly reduced.

<sup>3</sup> Von der Fehr and Harbord (1993) model the pool as a uniform-price, multi-unit auction. They show that the equilibria of this model do not necessarily lead to efficient dispatching with pool prices significantly above competitive levels. They also find that, in the oligopoly case, the expected pool price is lower in a more fragmented industry.

In section 3, we look at a more explicit model of electricity pools. We extend the basic model by assuming that there are two types of generating plants, low cost base-load and high cost peak-load. We show that there are equilibria of this model that involve the generators withholding capacity to ensure the marginal price is set by peak-load units which can be bid at higher prices. Evidence that such strategies are used in the England and Wales pool is given by Wolak and Patrick (1996)<sup>4</sup>. No such incentive exists under a discriminatory pricing rule and the discriminatory auction therefore does even better relative to the uniform auction than the basic model suggests.

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<sup>4</sup> Wolak and Patrick (1996) look at the time series properties of the 48 half-hourly prices using data from the pool between 1991-95. They argue that the empirical evidence is consistent with the generators withholding capacity and identify two reasons why this is profitable. The first is the reason identified above - to ensure the marginal price is set by peak-load units which can be bid at higher prices. The second is to ensure a high capacity charge (CC) which is paid in addition to the marginal price in the England and Wales pool. This payment increases rapidly as the difference between the forecasted demand and total quantity made available (the reserve margin) decreases. The capacity payment was clearly designed to encourage investment in capacity when the reserve margin was persistently low. However, the generators help determine CC through the capacity they make available and therefore have an incentive to withhold capacity whatever the auction design.

## 2. Basic Model

This section studies equilibria in multi-unit, common-value auctions, where there is a limit on the number of units each firm can bid for and the quantity for auction is uncertain<sup>5</sup>. We study the *reverse* auction case where suppliers compete to sell units of a good. The limit then represents the maximum number of units the firms have a capacity to supply. The results apply equally to conventional auctions where a limit may be imposed by the auctioneer.

There are  $m$  firms who each have enough capacity to supply  $k$  units. Normalise the total capacity of each firm to be 1 so the size of each unit is  $1/k$ . All firms produce at a constant marginal cost,  $c$ , up to capacity (this is the common value assumption). The firms submit bids for the minimum price at which they are willing to supply each of their units. Assume the maximum permissible bid is  $p^u$ . Let  $S(p)$  be the total capacity bid at or below  $p$ . After the firms submit their bids, nature chooses the level of demand,  $d$ . Let  $(\underline{d}, \bar{d})$  be the minimum and maximum values of demand. The firms know the demand distribution. The market clearing price,  $p(d)$ , is the lowest price such that  $S(p) \geq d$ . If more than one firm has bid units at  $p(d)$  and  $S(p(d)) > d$  then units bid at  $p(d)$  are rationed<sup>6</sup>. In cases where demand does not increase in increments of  $1/k$ , we assume a fraction of a unit is sold at the margin. The firms are risk-neutral and maximise expected profit. In this section, we study the equilibria of this model under a uniform and discriminatory auction format and compare the two auction formats in terms of the expected cost to the buyer. All proofs are given the appendix.

### Uniform-price auction

Under a uniform-price auction the sellers are paid the market clearing price,  $p(d)$ , for the units sold. This case is analysed by von der Fehr and Harbord (1993) when each firm has a different constant marginal cost. Many of their results are based on a proposition which only holds when marginal costs are different and so their results cannot be directly applied. However, the results we

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<sup>5</sup> Uncertain demand is motivated by the application to electricity pools.

<sup>6</sup> We assume any rationing rule where all these units have a positive probability of being sold.

present are similar in nature.

**Proposition 1:** *If  $Pr(d \leq m-1) = 1$  then there is a unique type of pure-strategy equilibrium where a quantity no less than  $\bar{d}$  is bid at marginal cost by the  $m-1$  firms excluding firm  $i$ , for all  $i$ .*

This is the standard Bertrand type result where undercutting ensures competitive prices. The crucial feature is that there is no residual market share as demand can be met by  $m-1$  firms.

**Proposition 2:** *If  $Pr(d > m-1) = 1$  then there is a unique type of pure-strategy equilibrium where one firm sets  $p^u$  for a quantity greater than  $m - \underline{d}$  and all other firms set a price sufficiently low that this firm cannot gain by lowering the price of these units.*

**Remark:** The simplest form of such an equilibrium is where one firm bids  $p^u$  for all  $k$  units and all other firms bid  $c$  for all  $k$  units. The high price firm only sells a quantity equal to  $d - (m-1)$  while all other firms sell  $k$  units.

This type of equilibrium is the worst possible for the buyer as all units are bought at the maximum (or *reserve*) price. The intuition behind this result is simple. When a firm faces a residual market, they know they will sell some units irrespective of their bids. However, in equilibrium, it is not possible to have more than one firm bidding at the maximum permissible price because the firms can gain by undercutting slightly as this avoids rationing. Hence, in equilibrium, one of the firms facing a residual market will bid at the maximum permissible price. The equilibrium requires that one firm is certain to have some residual market. We now look at the case where the probability that a firm has a residual market is positive but less than one. Let  $X(d)$  be any distribution function on the interval  $[\underline{d}, \bar{d}]$  where  $X(d_1) - X(d_2) > 0$  for all  $d_1, d_2$  such that  $\underline{d} \leq d_2 < d_1 \leq \bar{d}$ .

**Proposition 3:** *If  $\underline{d} < m-1 < \bar{d}$  then there is no pure-strategy equilibrium if demand is distributed according to  $X(d)$  or  $(\bar{d} - \underline{d})$  is less than 1.*

In the appendix, we show that it is possible to get equilibria of the type given in proposition 2 when the distribution has *holes* and  $(\bar{d} - \underline{d})$  is greater than 1. Where there are no pure-strategy equilibria, it may be possible to characterise mixed-strategy equilibria but this is not a simple exercise in the multi-unit case. An example of a mixed-strategy equilibrium where the firms can only set one price for all units is given by von der Fehr and Harbord (1993). However, in the appendix we show that this type of mixed-strategy equilibrium does not hold if each firm can submit an increasing supply schedule.

### **Discriminatory auction**

Under a discriminatory auction the sellers are paid the bid prices for the units they sell. We now characterise the equilibria under the discriminatory pricing rule. We begin by presenting a general result on the type of pure-strategy equilibria that can exist.

**Proposition 4:** *In any pure-strategy equilibrium the marginal price is  $c$ .*

This implies that under any pure-strategy equilibrium we always get the competitive outcome. The intuition is that if there is a bid above  $c$  that might be successful then the other firms will always have an incentive to undercut this bid slightly to ensure one of their units are sold instead. Competition to sell units eliminates any pure-strategy equilibrium where the marginal price is above marginal cost.

**Proposition 5:** *If  $\Pr(d \leq m-1) = 1$  then there is a unique type of pure-strategy equilibrium in which at least  $\bar{d} + 1$  units are bid at  $c$ .*

As in the uniform auction case we get the standard Bertrand type result when there is no chance that a firm will face some residual demand.



**Proposition 6:** *If  $\bar{d} > m-1$ , then there is no pure-strategy equilibrium.*

Hence, there is no pure-strategy equilibrium when there is a positive probability that a firm will have some residual market. This result is reminiscent of Edgeworth Cycles. The major difference with this model is that demand is infinitely inelastic<sup>7</sup> and the firms can price discriminate. For the remainder of this section, we concentrate on the duopoly case,  $m=2$ . Where there are no pure-strategy equilibria we look for mixed-strategy equilibria.

**Proposition 7:** *If  $Pr(d > 1) = 1$  then there is a mixed-strategy equilibrium in which each firm submits a price  $p \in (p_v, p^u)$  with probability  $f(p) = \frac{E(d)-1}{2-E(d)} \frac{(p^u - c)}{(p - c)^2}$  for all  $k$  units, where  $p_v = (E(d)-1)p^u + (2-E(d))c$  and  $E(d)$  is the expected value of demand.*

The expected cost to the buyer under the mixed-strategy equilibrium is

$$\begin{aligned}
 C &= \int_{p_v}^{p^u} \int_{p_1}^{p^u} (p_1 + (E(d)-1)p_2) f(p_2) dp_2 \\
 &\quad + \int_{p_v}^{p_1} (p_1(E(d)-1) + p_2) f(p_2) dp_2 \Big] f(p_1) dp_1 \quad (1) \\
 &= 2(E(d)-1)p^u + (2-E(d))c.
 \end{aligned}$$

This is the same expected cost that would arise if each firm is paid the maximum permissible price for  $(E(d)-1)$  units (the average amount by which demand exceeds capacity) and marginal cost for the rest. A similar result holds for the case where  $\underline{d} < 1 < \bar{d}$ . However, it does not hold for any demand distribution.

**Proposition 8:** *If  $\underline{d} < 1 < \bar{d}$  then there is a class of demand distributions for which there exists a mixed-strategy equilibrium in which each firm submits a price  $p \in (p_v, p^u)$  with probability*

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<sup>7</sup> Up to the maximum permissible price.

$$g(p) = \frac{(d^+ - 1)p^+}{(p^+(2 - d^+) + p^-d^-)} \frac{(p^u - c)}{(p - c)^2} \text{ for all } k \text{ units, where } p^+ = \Pr(d > 1), p^- = \Pr(d < 1), d^+ = E(d/d > 1),$$

$$d = E(d/d < 1) \text{ and } p_v = (p^+(d^+ - 1)p^u + (p^+(2 - d^+) + p^-d^-)c) / (p^+ + p^-d^-).$$

In the proof we give examples of demand distributions where this equilibrium does not hold. The examples require that the demand distribution be heavily skewed towards low demands. When this is the case, we are unable to characterise an equilibrium. We can however say that there is no pure-strategy equilibrium and no mixed-strategy equilibrium where the firms bid a single price for all units.

The expected cost to the buyer under the mixed-strategy equilibrium is,

$$\begin{aligned} C &= \int_{p_v}^{p^u} \left[ \int_{p_1}^{p^u} (p^+(p_1 + (d^+ - 1)p_2) + p^-d^-p_1)g(p_2)dp_2 \right. \\ &\quad \left. + \int_{p_v}^{p_1} (p^+(p_1(d^+ - 1) + p_2) + p^-d^-p_2)g(p_2)dp_2 \right] g(p_1)dp_1 \quad (2) \\ &= 2p^+(d^+ - 1)p^u + (p^+(2 - d^+) + p^-d^-)c. \end{aligned}$$

Once again it is as if each firm is paid the maximum permissible price for the amount by which demand exceeds capacity and marginal cost for the rest.

The above results give a clear ranking in terms of cost when the quantity for auction is certain. For simplicity we restrict attention to the duopoly case in the following discussion. When  $d < 1$  there is a unique type of pure-strategy equilibrium where the marginal price is always  $c$  under both pricing rules. Hence in each case the buyer simply pays marginal cost for all units. It is not surprising that we get this competitive Bertrand type result as each firm can supply the entire demand.

The most interesting case when capacity constraints are present is  $d > 1$ . Each firm is then sure to sell some units as the capacity of the other firm is 1. In the uniform-price auction case there is a unique set of pure-strategy equilibria where the marginal price is always  $p^u$  (from proposition 2). The cost for the buyer is then  $p^ud$ . Under a discriminatory auction there is no pure-strategy equilibrium for

this case (proposition 6). In the mixed-strategy equilibrium given in proposition 7, the cost to the buyer is  $2p^u(d-1)+(2-d)c$ . This is less than the cost under the pure-strategy equilibrium in the uniform case as  $c < p^u$ . In fact, we can show there is no equilibrium in the discriminatory case that results in a cost of  $p^u d$ . This would require both firms to bid  $p^u$  for all units (as they get paid their bids) but each firm can then gain by undercutting the other. Hence we have a clear ranking when each firm has some residual demand which makes the discriminatory auction less costly than the uniform one.

When demand is uncertain, the above results apply to the cases where  $\Pr(d < 1) = 1$  and  $\Pr(d > 1) = 1$  respectively. We simply replace cost by expected cost. For example, when  $\Pr(d > 1) = 1$ , the expected cost is  $2(E(d)-1)p^u + (2-E(d))c$  under the discriminatory auction and  $p^u E(d)$  under the uniform auction. The case where  $\underline{d} < 1 < \bar{d}$  is problematic as we were unable to characterise equilibria for the uniform-price auction case<sup>8</sup>. For the discriminatory auction we characterise a mixed-strategy equilibrium which holds for a large class of demand distributions. Hence we cannot rank the auctions in this case. The problem only arises if the quantity for auction is uncertain and sufficiently variable that each firm may or may not have some residual market. The motivation for uncertain demand comes from the application to electricity pools. In most other applications the quantity for auction is known. Also, in the next section we present a more explicit model of a pool and find that we can characterise equilibria in most cases.

The most important insight of this section is that the discriminatory auction has a definite advantage when there are capacity constraints to the extent that a firm faces some residual market. These results are of general theoretical interest. Much of the interest in multi-unit auctions stems from the sale of Treasury bonds where both discriminatory and uniform auctions have been used. There is a significant amount of empirical work comparing the two auctions but this has proved inconclusive<sup>9</sup>. Theoretical results comparing the auctions are limited. Back and Zender (1993) use the share auction framework of Wilson (1979), where the good is perfectly divisible and has a common value. They find that with a uniform auction, any price between the reservation price and the lower bound of the

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<sup>8</sup>It is possible to get pure-strategy equilibria of the type given in proposition 2 when the demand distribution has *holes* and  $(\bar{d} - \underline{d})$  is greater than 1. An example is given in the appendix after the proof of proposition 3.

common-value distribution can be supported as a symmetric Nash equilibrium. There is therefore a multiple-equilibrium problem and some of these equilibria are extremely bad for the seller. However, the multiple-equilibrium problem disappears when units are discrete as in our model<sup>10</sup>. Ausubel and Crampton (1995) concentrate on the relative efficiency of the auctions but also demonstrate that the revenue ranking is ambiguous.

It is clear that little can be said in general about revenue ranking in the multi-unit auction case. However, we are able to get a ranking in this specific model. The important feature of this model is that there are *binding* capacity constraints<sup>11</sup> which ensure that the firms face some residual market. Perhaps the best example of such a market in practice is an electricity pool where the constraints reflect the generating capacity of each firm. Such constraints should be present in any efficient generating system as if it is the case that demand can always be met by  $m-1$  firms then there is surely over-investment in generating stock. The model predicts that when the generators face some residual market share the discriminatory auction leads to more competitive prices. Intuitively, a firm with residual market share has complete control over the marginal price which determines the price paid for all units dispatched under a uniform pricing rule. A simplifying assumption of this model when applied to the electricity pool is that all firms have the same constant cost generating technology. We rest this assumption in the next section by assuming there are two types of constant cost technology, low cost and high cost.

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<sup>9</sup> See Binmore and Swierzbinski (1997) for a survey.

<sup>10</sup> See for example Anwar (1998) chapter 3.

<sup>11</sup> Both Ausubel and Crampton and Back and Zender extend their results to the case where there are capacity constraints but only to the extent that there is always competition for each unit. By binding capacity constraints we mean capacity constraints which leave some residual market for one of the bidders.

### 3. Electricity Pool Model

The model in the previous section illustrates how market power arises from a uniform pricing rule when the firms face residual demand. In this section we extend the basic model by assuming there are two types of generating plant, low cost base-load and high cost peak-load. We show how this leads to an additional source of market power under a uniform pricing rule. The basic intuition is that peak-load units can justifiably be bid at much higher prices than base-load units. This gives the generators an incentive to withhold base-load capacity to increase the probability that the marginal price is set by peak-load units. The assumption that the maximum permissible bid for a particular unit depends on marginal cost is crucial. In practice there are no explicit restrictions on bids. However, if bids are persistently well in excess of marginal cost, the government will have ample evidence to make a case of market power to the competition authority. Since a necessary feature of generation is plant maintenance it would be more difficult to make a case of market power on the grounds that the generators are withholding capacity.

#### Model

For simplicity we assume there are only two firms that compete to supply electricity to the pool. Each firm owns a number of generating plants. The firms submit bids for the minimum price at which they are willing to supply electricity from each of their plants. In addition they also submit bids for the generating capacity they wish to make available to the pool. This may involve taking out an entire plant or only making part of a plant available. The bids are used to construct the aggregate supply schedule. The marginal price (MP) is the bid price of the marginal unit that is required to meet the realised level of demand. Each generator dispatches the units that were bid below MP plus some or all that were bid at MP<sup>12</sup>. Demand is distributed according to  $P(d)$ ,  $d \in [\underline{d}, \bar{d}]$ , and this is common knowledge.

There are two types of generating plant, low-cost base-load and high-cost peak-load, which

have marginal cost  $c_b$  and  $c_p$  respectively. Price bids cannot be more than  $m$  units above marginal cost. Each firm owns  $k$  units of base-load plants. As in the basic model we normalise the base-load capacity of each firm to 1. We assume that only firm 2 owns peak-load plants<sup>13</sup> and that  $c_p > m + c_b$ <sup>14</sup>. Hence the marginal cost of the peak-load units is greater than the maximum permissible base-load price. If insufficient base-load capacity is made available, the marginal price is set by high-cost, peak-load units. We begin by characterising the price equilibria given the capacity choices. Let  $(y_1, y_2)$  be the base-load capacity made available by the two firms. Then we have the following equilibria in prices.

### Equilibria in prices.

In equilibrium, firm 2 will never set a price below  $c_p$  for peak-load units as they will make a loss in the event these units are dispatched. Also, since  $c_p > m + c_b$ , firm 2 cannot undercut a base-load unit with a peak-load unit. The strategic use of peak-load units comes from the fact that the marginal price is high when it is set by peak-load units. Hence firm 2 will set the maximum price  $c_p + m$  for all peak-load units. We now consider equilibria in base-load prices.

**Proposition 9:** *If  $\Pr(d < \min\{y_1, y_2\}) = 1$  then there is a unique type of pure-strategy equilibrium where the marginal price is always  $c_b$ .*

This follows from proposition 1. When firms have different capacities, it is necessary for demand to be less than the capacity of each firm to get the competitive outcome.

**Proposition 10:** *If  $y_1 + y_2 > \underline{d} > \min\{y_1, y_2\}$  then there is a unique type of pure-strategy equilibrium, where one firm sets the maximum permissible price for a quantity greater than  $y_1 + y_2 - \underline{d}$  and the other firm sets a price sufficiently low so that this firm cannot gain by lowering the price of these units.*

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<sup>12</sup> In the event that units are rationed, they all have a positive probability of being dispatched.

<sup>13</sup> We make this assumption to avoid complications from mixed strategy peak-load prices. The focus of this model is on capacity choices and competition to dispatch base-load units.

<sup>14</sup> This assumption is made to separate base-load and peak-load competition. Once again the focus is on base-load competition, the role of peak-load units is to ensure a high MP.

This follows from proposition 2. If  $\min\{y_1, y_2\} < \underline{d} < \max\{y_1, y_2\}$  then the firm with the larger capacity must be the high price firm.

### Quantity choices.

For a given demand distribution it is clear why a capacity withholding strategy is profitable. Restricting the amount of capacity made available will increase the chance that the marginal price is set at  $c_p+m$ . This is illustrated in figure 1. However, the desire to withhold capacity must be balanced against the profitability of making a surplus on units not made available. The equilibrium will involve a balance between these opposing forces.

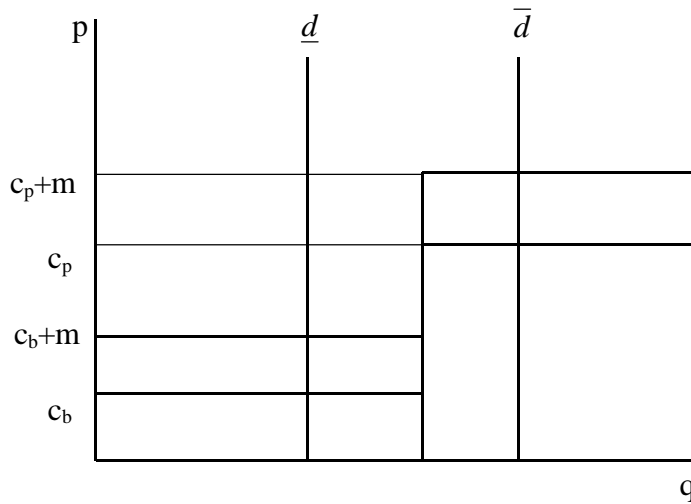


Figure 1  
Quantity choices

### Equilibria in prices and quantity.

To investigate equilibria in prices and quantity we analyse the model with a uniform demand distribution. From proposition 9 when  $\min(y_1, y_2) < \underline{d}$  the equilibrium in prices will involve one firm setting a low price and the other setting the maximum  $c_b+m$ . In this type of equilibrium let firm 1 be the low price firm submitting all units at marginal cost and firm 2 the high price firm submitting all units at  $c_b+m$ . The condition for this equilibrium then becomes  $y_1 < \underline{d}$ . For every equilibrium we

characterise where firm 1 is the low price firm there is a similar<sup>15</sup> equilibrium where firm 2 is the low price firm. We begin by characterising the equilibria when there are no binding capacity constraints. Let  $\bar{d} - \underline{d} = D_d$  and  $c_p - c_b = D_c$ .

The profit functions conditional on  $\underline{d} < y_1 + y_2$  are,

$$\begin{aligned} p_1 &= \frac{\bar{d} - y_1 - y_2}{\bar{d} - \underline{d}} (\Delta_c + m) y_1 + \frac{y_1 + y_2 - \underline{d}}{\bar{d} - \underline{d}} m y_1, \\ p_2 &= \int_{y_1 + y_2}^{\bar{d}} \frac{(d - y_1 - y_2)m}{\bar{d} - \underline{d}} dd + \frac{\bar{d} - y_1 - y_2}{\bar{d} - \underline{d}} (\Delta_c + m) y_2 + \int_{\underline{d}}^{y_1 + y_2} \frac{(d - y_1)m}{\bar{d} - \underline{d}} dd \\ &= \frac{m}{\bar{d} - \underline{d}} \left( \frac{\bar{d}^2}{2} - (y_1 + y_2)\bar{d} + \frac{(y_1 + y_2)^2}{2} \right) \\ &\quad + \frac{\bar{d} - y_1 - y_2}{\bar{d} - \underline{d}} (\Delta_c + m) y_2 + \frac{m}{\bar{d} - \underline{d}} \left( \frac{(y_1 + y_2)^2}{2} - (y_1 + y_2)y_1 - \frac{\underline{d}}{2} + \underline{d}y_1 \right). \end{aligned}$$

Both profit functions are concave given  $D_c > 0$ . From the first order condition we get the following reaction functions

$$\begin{aligned} \bar{R}_1(y_2) &= \frac{\bar{d}\Delta_c - \Delta_d m}{2\Delta_c} - \frac{1}{2}y_2, \\ \bar{R}_2(y_1) &= \frac{\bar{d}}{2} - \frac{1}{2}y_1. \end{aligned} \tag{3}$$

The solution is

$$\begin{aligned} y_1^* &= \frac{\bar{d}\Delta_c + 2\Delta_d m}{3\Delta_c}, \\ y_2^* &= \frac{\bar{d}\Delta_c - \Delta_d m}{3\Delta_c}. \end{aligned} \tag{4}$$

The profit functions conditional on  $\underline{d} > y_1 + y_2$  are

$$\begin{aligned} p_1^b &= (c_p + m)y_1, \\ p_2^b &= (c_p + m)y_2 + (E(d) - y_1 - y_2)m. \end{aligned}$$

These are strictly increasing in quantity. Hence, if  $\underline{d} < y_1 + y_2$ , firm i will want to set quantity to at least

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<sup>15</sup> But not symmetric, as firm 2 owns peak-load units.



$\underline{d} - y_j$ . It will set quantity to  $\bar{R}_i(y_j)$  if  $\bar{R}_i(y_j) > \underline{d} - y_j$ . The reaction functions are therefore

$$\begin{aligned} R_1(y_2) &= \max[\bar{R}_1(y_2), (\underline{d} - y_2)], \\ R_2(y_1) &= \max[\bar{R}_2(y_1), (\underline{d} - y_1)]. \end{aligned} \tag{5}$$

**Proposition 11:** *There is a set of equilibria where each firm bids at least  $\bar{d}$  at marginal cost.*

This follows from proposition 9 when both firms make enough capacity available to cover demand. The unique equilibrium in prices involves bidding at least  $\bar{d}$  at marginal cost. Neither firm can gain by withholding capacity if the other firm has bid  $\bar{d}$  at marginal cost.

**Proposition 12:** *If  $\underline{d} > \frac{\Delta_d (2\Delta_c + m)}{\Delta_c}$  then there is a continuum of equilibria where  $y_1 + y_2 = \underline{d}$  irrespective of the base-load price bids.*

This type of equilibrium is illustrated in figure 2. Hence, if the level of demand is high enough holding  $\Delta_d$  fixed, there are capacity withholding equilibria where the marginal price is always set by peak-load units. The critical value of  $\underline{d}$  depends on the difference between the marginal cost of the peak-load and base-load units,  $\Delta_c$ . The greater this difference the lower the critical value.

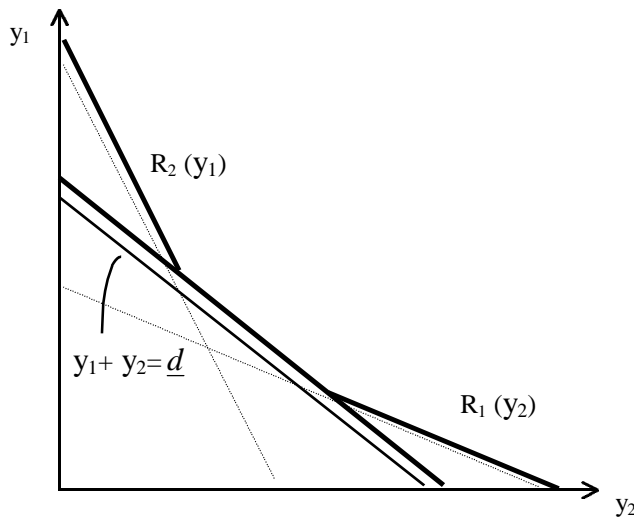


Figure 2.  
Continuum of equilibria.

**Proposition 13:** If  $\frac{\Delta_d(\frac{1}{2}\Delta_c + m)}{\Delta_c} < \underline{d} < \frac{\Delta_d(2\Delta_c + m)}{\Delta_c}$  then there is an equilibrium in prices and quantities where  $y_1=y_1^*$ ,  $y_2=y_2^*$  and firm 1 submits base-load units at marginal cost and firm two sets the maximum price,  $m+c_b$ .

This equilibrium is illustrated in figure 3. The expected pool price in this equilibrium is  $\frac{\bar{d} - y_1^* - y_2^*}{\Delta_d}(c_p + m) + \frac{y_1^* + y_2^* - \underline{d}}{\Delta_d}(c_b + m)$ . The total base-load capacity made available under this equilibrium,  $y_1^* + y_2^*$ , is decreasing in  $\Delta_c$ . The expected pool price is therefore increasing in  $\Delta_c$ .

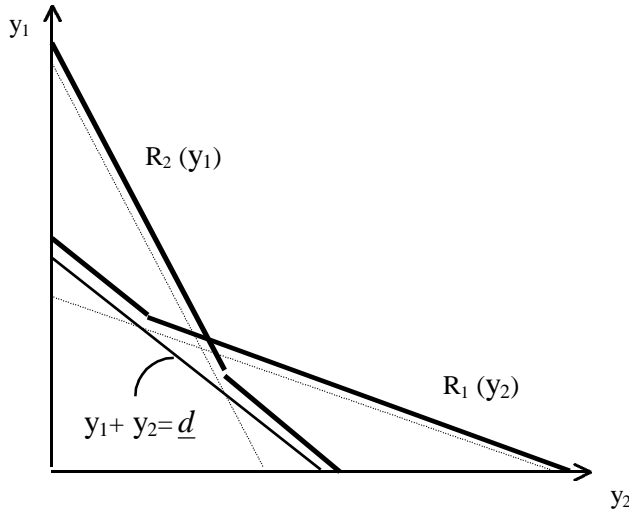


Figure 3  
Unique equilibrium

If  $\underline{d} < \frac{\Delta_d(\frac{1}{2}\Delta_c + m)}{\Delta_c}$  then the base-load price equilibrium breaks down as  $y_1^* > \underline{d}$ . This is illustrated in figure 4. The equilibrium given in proposition 11 still applies if the firms make enough capacity available. There is also an incentive to withhold capacity, although there is no pure-strategy equilibrium in base-load prices if capacity is withheld. There may be mixed-strategy equilibria in base-load prices. In any such equilibrium, the expected base-load price must lie somewhere between  $c_b$  and  $c_b+m$ . An expected base-load price below  $c_b+m$  will only increase the incentive to withhold capacity as this would increase the gap between the peak-load price and base-load price. So although we are unable to characterise a capacity withholding equilibrium in this case, capacity withholding to increase

the chance of obtaining a high marginal price is clearly profitable.

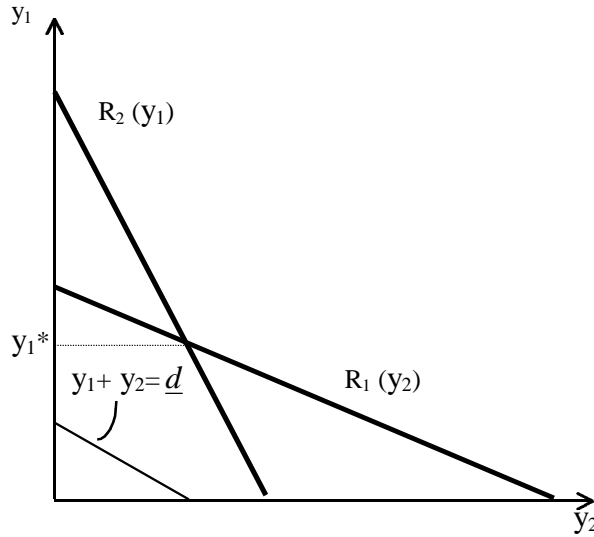


Figure 4.  
No base-load price equilibrium.

The above analysis ignores capacity constraints. Since each firm has a total base-load capacity of 1, the equilibria given in proposition 11 only hold when  $\bar{d} < 1$  as it is necessary for each firm to bid at least  $\bar{d}$  at marginal cost. Now consider the capacity withholding equilibria with the capacity constraint of 1. If demand is so high that  $y_1^* > 1$  or  $y_2^* > 1$  then the constrained firm will set  $y_i = 1$  (since the profit function is concave and therefore increasing in quantity when  $y_i < y_i^*$ ). The other firm will then set  $\min(R_j(1), 1)$ . This will result in a greater probability of marginal price being set by the peak-load units. Hence, constraints on base-load capacity will result in a higher average pool price than the above analysis suggests.

As in the basic model, it is the control over the marginal price that gives the firms market power. The difference with this model is that the marginal price is controlled by adjusting the base-load capacity made available. By withholding capacity, the firms can increase the chance that the marginal price is set by peak-load units which are bid at much higher prices. However, capacity constraints still have a role to play as it is a combination of the limited base-load capacity and the deliberate withholding of some of this capacity that allows the marginal price to be set by peak-load units.

### **Discriminatory pricing rule.**

We now consider the equilibria of this model under a discriminatory pricing rule and compare the outcomes with the uniform-price model. Although it would be straightforward to allow both firms to have peak-load units we stick to the case where only firm 2 has peak-load units for consistency. Under a discriminatory pricing rule there is no incentive to manipulate the marginal price as the generators are simply paid their bid prices for the units that are dispatched. Also, firm 2 does not gain by submitting peak-load units rather than base-load units, as the maximum profit they can make on any unit is  $m$ , irrespective of the marginal cost. Withholding capacity will only decrease the expected quantity dispatched. The model for base-load units is then formally equivalent to the common value model of section 2.

### **Price equilibria.**

Peak-load units will never be bid below  $c_p$  as firm 2 would make a loss on these units. Therefore in equilibrium, peak-load units are only dispatched when the demand is greater than the total base-load capacity. As in the previous model, firm 2 will therefore set the maximum price,  $c_p+m$  for all peak-load units<sup>16</sup>.

The base-load price equilibria correspond to those given in section 2. If  $\bar{d} < 1$  then there is a unique pure-strategy equilibrium where firms submit base-load units at marginal cost. The uniform price model has a similar equilibrium (proposition 11) but there is also a capacity withholding equilibrium where the average pool price lies somewhere between  $c_b+m$  and  $c_p+m$ . Hence, unlike in the basic model, the uniform pricing rule does not necessarily lead to competitive prices when demand is less than the capacity of one of the firms whereas the discriminatory pricing rule does. If  $\bar{d} > 1$  then there is no pure-strategy equilibrium in base-load prices (proposition 6). There is a mixed-strategy

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<sup>16</sup> If both firms have peak-load units then the equilibria in peak-load prices are formally the same as the base-load equilibria.

equilibrium where each firm submits one price for all units (proposition 8)<sup>17</sup>.

It is simple to see that the expected cost in the capacity withholding equilibrium is always greater than under the mixed-strategy equilibrium of the discriminatory auction. All peak-load units are sold at  $m+c_p$  in both cases<sup>18</sup>. However, under the capacity withholding equilibrium, all of the base-load units are sold at a cost of at least  $m+c_b$ , the maximum permissible base-load price, and some are sold at  $m+c_p$ , the maximum permissible peak-load price, whereas under a discriminatory auction, all the base-load units are sold at a price less than or equal to  $m+c_b$ <sup>19</sup>. Note also that the difference between the two pricing rules is greater than in the basic model. Whereas the base-load equilibria under the discriminatory pricing rule are unchanged, the uniform pricing rule leads to an average pool price above  $m+c_b$ , the maximum permissible base-load bid.

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<sup>17</sup> If  $\underline{d} < 1$  then this equilibrium holds as long as the demand distribution is not heavily skewed to the left. See section 2 for details. We need to make one change to the notation in proposition 8 to account for the case where  $\bar{d} \geq 2$ ,

$$d^+ = \frac{\Pr(1 < d < 2)(E(d / 1 < d < 2) + 2 \Pr(d \geq 2))}{\Pr(d > 1)}.$$

<sup>18</sup> To keep things tractable in the uniform auction case we eliminated any competition to dispatch peak-load units by assuming that only firm 2 owned peak-load units.

<sup>19</sup> The price will be equal to  $m+c_b$  if  $\underline{d} > 2$ .

#### 4. Discussion

The pool model is admittedly simple but it does demonstrate some important features of the two pricing rules. The most important feature of the uniform pricing rule is that only the marginal price matters in determining the price paid for units dispatched. As a result, generators have complete control over the price paid for all units dispatched when there is no competition at the margin (i.e. when a firm has some residual market). In fact, the model demonstrates that a residual market is not necessary to get these high prices. If the firms feel constrained in their base-load bids, there are capacity withholding equilibria even when each firm can satisfy the entire demand! In practice the generators will not get away with using the capacity withholding strategy indiscriminately. However, the model does highlight an additional weapon the generators have at their disposal to secure high pool prices under a uniform pricing rule. The discriminatory pricing rule does not suffer from these drawbacks. The marginal price does not have the same level of significance as it only determines the price paid to the marginal unit. Hence there is no incentive to withhold capacity to ensure a high marginal price.

A feature of the England and Wales pool not modelled here is financial contracts between the generators and the RECs which essentially fix the price of the contracted quantity. Such contracts are likely to arise when pool prices are uncertain and volatile. In Anwar (1998) we extend the pool model to include contracts for differences and show that this does not eliminate the incentive to withhold base-load capacity. Intuitively, the firms are competing to supply the uncovered market where the marginal price determines the price paid for units dispatched. Hence the arguments of sections 2 and 3 can be applied to the uncovered part of the market. Furthermore, one must recognise that the contract market and the pool are interdependent. The contract strike price<sup>20</sup> will be related to the average pool price and price volatility. Our model of the pool predicts extremely volatile pool prices under a uniform pricing rule with prices varying from peak-load to base-load. A pool with volatile prices which are also high on average will give rise to a contract market with high strike prices. Under a discriminatory pricing rule, not only is the volatility significantly reduced with base-load prices varying

over a much smaller range, the average pool price is also significantly more competitive. This would result in contracts with strike prices set at more competitive rates.

A question we do not address in our model is the relative efficiency of the two auctions. There is a misguided view that the uniform auction format is efficient<sup>21</sup>. One of the main factors behind the reluctance to use a discriminatory format in the England and Wales pool is that it is thought to be inefficient<sup>22</sup>. Ausubel and Crampton (1995) look at the efficiency properties of the auctions in general. They prove an inefficiency theorem for the uniform-price auction which applies when there is a private-values component and bidders demand more than one unit<sup>23</sup>. The inefficiency arises from the fact that large bidders will shade more than small bidders and sometimes lose units to small bidders who actually value them less. Although our model of the pool is too simplistic to do an efficiency comparison it does demonstrate a great inefficiency that arises in a uniform auction with peak-load units being dispatched when base-load units are available.

An alternative pricing mechanism that is often considered in the theoretical literature is the multi-unit Vickrey auction<sup>24</sup>. The multi-unit Vickrey pricing rule is as follows. Each firm dispatches units bid at or below the marginal price. For the  $n$ th unit dispatched, the firm is paid a marginal price that is calculated by looking at the intersection of the supply of the other firms with demand net of the capacity of  $n-1$  units. Hence, for the first unit dispatched, the firm is simply paid the marginal price that would arise if none of its units are made available, for the second unit dispatched the firm is paid the marginal price that would arise if demand was then reduced by the capacity of the first unit and so on. Although the rule is a bit more complicated, the firms should soon learn that it is a weakly dominant strategy to simply bid at marginal cost<sup>25</sup>. This truthful revelation of cost is a desirable quality in general as it ensures efficiency. However, in the electricity pool model the compensation required to elicit the truthful revelation is great. In particular, whenever demand is greater than 1 in our model (or

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<sup>20</sup> The price at which the contracted quantity is effectively traded.

<sup>21</sup> See Ausubel and Crampton (1995) for a discussion.

<sup>22</sup> See for example Offer (1998), Review of Electricity Trading Arrangements.

<sup>23</sup> Or supply more than one unit in the reverse auction case.

<sup>24</sup> Von der Fehr and Harbord (1993) consider using the Vickrey auction in their model of the pool but make a slight mistake in the multi-unit extension.

<sup>25</sup> See Vickrey (1961) page 11 or more specifically Anwar (1998) page 151.

the base-load capacity of one of the firms in general), the price paid to marginal units will be the marginal cost of the peak-load units. Hence peak-load prices will be paid even when no peak-load capacity is required. Also, the Vickrey rule can result in even higher payments as the firms can easily collude on weakly dominated equilibria from which no firm will have even a short-run incentive to deviate. The problem is that it is only a weakly dominant strategy to bid at marginal cost. Increasing some bids above marginal cost will benefit other firms without making the firm any worse off.



## Conclusions

Following the England and Wales lead, competitive electricity pools are being adopted around the world as a means to introducing competition in electricity generation. An important lesson from the England and Wales experience has been the importance of limiting market concentration and hence market power. We look at the role played by the pricing rule in determining the extent of market power. We have shown that for any given number of firms, the uniform pricing rule leads to market power when demand is sufficiently high. The market power arises from control over the marginal price which determines the price paid for all units dispatched. In addition, we identify an incentive to withhold base-load capacity under a uniform pricing rule when there are regulatory constraints on bids. Withholding capacity increases the chance that the marginal price is set by (high cost) peak-load units which can be bid at much higher prices.

The discriminatory pricing rule does not suffer from these drawbacks. The marginal price is not as significant as it only determines the price paid to the marginal unit. Hence there is no incentive to withhold capacity and ensure the marginal price is high. We show that the equilibria with a discriminatory pricing rule result in lower pool prices. We therefore put a strong case for a discriminatory auction to be used in competitive electricity pools in preference to a uniform auction.

## Appendix

**Lemma 1:** *Under a uniform pricing rule, there is no pure-strategy equilibrium where  $p'' > p(\bar{d}) > c$ .*

**Proof of Lemma 1:** Any unit bid above  $p(\bar{d})$  has a zero probability of being successful. If more than one firm is bidding at or above  $p(\bar{d})$  then at least one firm can gain by changing one of these bids to just below  $p(\bar{d})$  as the unit will then have a positive probability of being successful at a profit and it avoids rationing. If only one firm has units at or above  $p(\bar{d})$  then that firm can gain by raising these bids to  $p''$ .

**Lemma 2:** *Under a uniform pricing rule, there is no pure-strategy equilibrium where  $p'' > p(\underline{d}) > c$ .*

**Proof of Lemma 2:** Assume by contradiction that there is a pure-strategy equilibrium where  $p'' = p(\bar{d}) > p(\underline{d}) > c$ <sup>26</sup>. We begin by assuming there are two firms with bids in the interval  $[p(\underline{d}), p'']$ . If only one firm has units bid at  $p(\underline{d})$  then this firm can gain by increasing these bids towards the next highest bid of another firm. If more than one firm has bids equal to  $p(\underline{d})$  then each firm can gain by reducing the bids slightly to avoid rationing. Finally, if only one firm has bids in the interval  $[p(\underline{d}), p'']$  then that firm can gain by increasing these bids to  $p''$ .

**Proof of proposition 1:** It is simple to see that this is an equilibrium as, if any firm raises their bids above marginal cost these units will have a zero probability of being sold. We now show that there is no other type of pure-strategy equilibrium. From lemma 1,  $p(\bar{d}) = c$  or  $p(\bar{d}) = p''$  in any pure-strategy equilibrium. However,  $p(\bar{d}) = p''$  requires more than one firm to bid at  $p(\bar{d})$  as the capacity of each firm is 1 and  $\bar{d} < m-1$ . These firms can gain by undercutting  $p''$ , thereby avoiding rationing. If  $p(\bar{d}) = c$  then the only way a firm can gain is by raising  $p(\bar{d})$ , but they cannot do this if at least  $\bar{d}$  is submitted at  $c$  by all the other firms. QED.

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<sup>26</sup> From lemma 1,  $p(\bar{d}) = c$  or  $p(\bar{d}) = p''$  in any pure-strategy equilibrium. Hence if there is an equilibrium where

**Proof of proposition 2:** If  $p(\underline{d})=c$  each firm can increase the marginal price in the event demand is  $\underline{d}$  by increasing the bids currently at  $c$  as  $\underline{d} > m-1$ . Hence, if  $p(\underline{d})=c$ , each firm can gain by increasing bids currently at  $c$  towards the next highest bid, as this will only affect the ranking in the event where they were previously making no profit and will result in a positive profit when demand is  $\underline{d}$ . There is therefore no equilibrium where  $p(\underline{d})=c$ .

From lemma 1 and lemma 2 the only remaining candidate for a pure-strategy equilibrium is  $p(\underline{d})=p(\bar{d})=p^u$ . However, it is not possible to have an equilibrium where more than one firm has bid units at  $p^u$  as the firms can undercut  $p^u$  slightly and avoid rationing. That leaves one possibility for an equilibrium - one firm bids enough units at  $p^u$  to ensure the marginal price is certain to be  $p^u$ . All the other firms will be indifferent between setting prices in the interval  $[c, p^u)$  as they sell  $k$  units for  $p^u$ . However, they must set prices sufficiently low such that the high price firm cannot gain by lowering prices and increasing the quantity sold. QED.

**Proof of proposition 3:** It is not possible to have an equilibrium where more than one firm has bid units at  $p^u$  as the firms can gain by undercutting. Since  $\underline{d} < m-1$ , it is not possible to have an equilibrium where  $p(\underline{d})=p^u$  as this would require more than one firm to submit units at  $p^u$ . Also, since  $m-1 < \bar{d}$  it is not possible to have an equilibrium where  $p(\bar{d})=c$  as each firm can make a positive profit by increasing bids. From lemma 1 and lemma 2 the only other candidate for an equilibrium is  $p(\underline{d})=c$  and  $p(\bar{d})=p^u$  with only one firm bidding at  $p^u$ . We now show that this is not possible if demand is distributed according to  $X(d)$  or  $(\bar{d} - \underline{d})$  is less than 1.

Assume by contradiction that we have such an equilibrium and demand is distributed according to  $X(d)$ . Firms who have bid units at  $c$  can gain by increasing these bids because such units will have a positive chance of becoming marginal as every interval of demand has a positive probability of occurring. This only affects the ranking in the event where they were previously making no profit and ensures a positive profit when one of these units set the marginal price.

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$p^u > p(\underline{d}) > c$  then  $p(\bar{d})$  must be equal to  $p^u$ .

Now assume by contradiction that we have such an equilibrium and  $(\bar{d} - \underline{d})$  is less than 1. The firm with units at  $p^u$  must have more than  $m - \bar{d}$  units at this price to ensure  $p(\bar{d}) = p^u$ . If any other firm increases bids on all units (capacity 1) this will ensure  $p(\underline{d})$  increases if  $m - \bar{d} + 1 \geq m - \underline{d}$  which requires  $\bar{d} - \underline{d} \leq 1$ . Increasing  $p(\underline{d})$  in this way will increase profits in the event demand is  $\underline{d}$ . QED.

**Example of a pure-strategy equilibrium when  $\underline{d} \leq m-1 < \bar{d}$ .**

Consider the following discrete example in the duopoly case. Demand takes two values with positive probability, 0.8 and 1.95, so the distribution has a hole and  $(\bar{d} - \underline{d})$  is greater than 1. Assume also that  $k=0.1$ . Then there is a pure-strategy equilibrium where one firm bids one unit at  $p^u$  and all others at  $c$  and the other firm bids  $c$  for all units. The part of the proof of proposition 3 that breaks down is that firms cannot gain by raising bids above marginal cost when  $p(\underline{d}) = c$  and this is because there is a hole in the distribution.

**Mixed-strategy equilibria when  $\underline{d} \leq m-1 < \bar{d}$ .**

It is clear that for a large class of distributions there is no pure-strategy equilibrium when  $\underline{d} \leq m-1 < \bar{d}$ . We are unable to find any mixed-strategy equilibria for these cases. If the firms are only allowed to set one price for all units then it is a simple exercise to find a mixed-strategy equilibrium. An example of such an equilibrium is given by von der Fehr and Harbord (1993). They assume that demand is discrete and takes two values, 1 and 2, with probabilities  $r$  and  $1-r$ . From proposition 3 there is no pure-strategy equilibrium in this case. They derive a mixed-strategy equilibrium by finding the distribution function the other firm needs to use for each firm to be indifferent between bidding a price in the interval  $[p_v, p^u]$ . Let the corresponding density function for firm 2 be  $t(p)$ . We now show that this equilibrium does not hold if firm 1 can split his unit in two and set two prices. The profit of firm 1 if he sets two prices and firm 2 is using the mixed strategy is,

$$\begin{aligned}
p_1(p_1, p_2) &= (1-r) \left( \int_{p_1}^{p^u} (p-c)t(p)dp + \int_{p_v}^{p_1} (p_1-c)t(p)dp \right) \\
&\quad + r \left( \int_{p_1}^{p^u} (p_1-c)t(p)dp + \int_{p_2}^{p_1} \frac{1}{2}(p-c)t(p)dp \right) \\
&= p_1(p_1) + r \int_{p_2}^{p_1} \frac{1}{2}(p-c)t(p)dp.
\end{aligned}$$

Hence the profit is equal to the profit the firm would get by setting one price plus some positive amount.

**Proof of proposition 4:** Let  $p^1$  be the lowest bid and  $p^i$  be the highest bid less than  $p(\bar{d})$ . If  $p(\bar{d}) = p^1 > c$  then any firm can gain by reducing their bids slightly below  $p(\bar{d})$ . This increases the expected number of units dispatched (as it avoids rationing) without significantly affecting the price. Now assume the aggregate supply is increasing. If only one firm has bid at  $p^i$ , then that firm can gain by increasing the price towards  $p(\bar{d})$ . If two or more firms have bid a unit at this price and these units are rationed with a positive probability, then these firms can gain by reducing their bids slightly and thereby increasing the expected quantity dispatched. If the units are not rationed then the firms can gain by raising these bids towards  $p(\bar{d})$ . Hence there is no equilibrium in which  $p(\bar{d}) > c$ . QED.

**Proof of proposition 5:** It is straightforward to see this is an equilibrium. Reducing the price below marginal cost will result in negative profits. Raising price above marginal cost will result in a zero probability of being successful. From proposition 4 there is no pure-strategy equilibrium where the marginal price is above  $c$ . QED.

**Proof of proposition 6:** From proposition 4 there is no pure-strategy equilibrium where the marginal price is greater than  $c$ . Hence in any pure-strategy equilibrium profits must be equal to 0. However, each firm can make a positive profit by setting a positive price for all units, as they will be dispatched in the event that demand is greater than  $m-1$ . QED.

**Proof of proposition 7:** Suppose firm 2 is submitting a price  $p \in (p_v, p^u)$  for all units, according to the distribution function  $F(p)$ . Let  $f(p)$  be the corresponding density function. Then player 1's expected payoff from submitting a price  $p_1$  for all  $k$  units is

$$\Pi(p_1) = \int_{p_1}^{p^u} (p_1 - c) f(p) dp + \int_{p_v}^{p_1} (p_1 - c) (E(d) - 1) f(p) dp.$$

In equilibrium  $\pi'(p_1)=0$  for all  $p_1 \in (p_v, p^u)$ . This gives

$$pf(p) + F(p) = \frac{1}{(2 - E(d))}.$$

The unique solution of this differential equation with boundary condition  $F(p^u)=1$ <sup>27</sup> is

$$f(p) = \frac{E(d) - 1}{2 - E(d)} \frac{(p^u - c)}{(p - c)^2},$$

$$F(p) = \frac{p}{(2 - E(d))(p - c)} - \frac{(E(d) - 1)p^u + (2 - E(d))c}{(2 - E(d))(p - c)}. \quad (6)$$

Solving  $F(p_v)=0$  gives  $p_v = (E(d)-1)p^u + (2-E(d))c$ . Hence given firm 2 is using the mixed strategy, firm 1 is indifferent as regards submission of any price in the interval  $[p_v, p^u]$  for all  $k$  units. We now show that, given firm 2 is using this mixed strategy, firm 1 cannot gain by submitting an increasing supply function. To do this we need to introduce some further notation.

Let  $P$  be a vector of  $n$  prices,  $\{p_1, \dots, p_n\}$  where  $n \leq k$ ,  $p_1 > p_2 > \dots > p_n$ ,  $p_1 \leq p^u$ ,  $p_n \geq p_v$ <sup>28</sup>. Let  $a_i$  be the number of units bid at  $p_i$  multiplied by  $1/k$ . Hence  $\sum_{i=1}^n a_i = 1$ . Let  $pr^i$  be the probability that

$2 - \sum_{j=0}^{i-1} a_j > d > 2 - \sum_{j=0}^i a_j$  and  $d^i$  be,  $E(d / (2 - \sum_{j=0}^{i-1} a_j > d > 2 - \sum_{j=0}^i a_j))$ , where  $a_0 = 0$ . The expected

profit of firm 1 as a function of the price vector is,

<sup>27</sup> It is not possible to have a mixed-strategy equilibrium with an upper bound less than  $p^u$  as the same quantity will be assigned by setting a price  $p^u$  as this upper bound but the price received is greater.

<sup>28</sup> Any bid below  $p_v$  cannot be optimal as firm 1 will be assigned such units with probability 1 but can increase the payment received for these units by increasing the bid to  $p_v$ .

$$\begin{aligned}
p(P) = & pr^1 \left( \int_{p_1}^{p^u} \sum_{i=1}^n a_i(p_i - c) f(p) dp \right. \\
& + \int_{p_v}^{p_1} \sum_{i=2}^n a_i(p_i - c) + (d^1 - (2 - a_1))(p_1 - c) f(p) dp \Big) \\
& + pr^2 \left( \int_{p_1}^{p^u} \sum_{i=1}^n a_i(p_i - c) f(p) dp + \int_{p_2}^{p_1} \sum_{i=2}^n a_i(p_i - c) f(p) dp \right. \\
& + \int_{p_v}^{p_2} \sum_{i=3}^n a_i(p_i - c) + (d^2 - (2 - a_1 - a_2))(p_2 - c) f(p) dp \Big) \\
& \vdots \\
& + pr^l \left( \int_{p_1}^{p^u} \sum_{i=1}^n a_i(p_i - c) f(p) dp + \int_{p_l}^{p_1} \sum_{i=2}^n a_i(p_i - c) f(p) dp + \right. \\
& + \sum_{m=3}^l \left( \int_{p_m}^{p_{m-1}} \sum_{i=m}^n a_i(p_i - c) f(p) dp \right) \\
& + \int_{p_v}^{p_l} \sum_{i=l+1}^n a_i(p_i - c) + (d^l - (2 - \sum_{j=1}^l a_j))(p_l - c) f(p) dp \Big) \\
& \vdots \\
& + pr^n \left( \int_{p_1}^{p^u} \sum_{i=1}^n a_i(p_i - c) f(p) dp + \int_{p_n}^{p_1} \sum_{i=2}^n a_i(p_i - c) f(p) dp \right. \\
& + \sum_{m=3}^n \left( \int_{p_m}^{p_{m-1}} \sum_{i=m}^n a_i(p_i - c) f(p) dp \right) \\
& + \int_{p_v}^{p_n} (d^n - 1)(p_n - c) f(p) dp \Big).
\end{aligned}$$

The first line is the expected profit conditional on  $2 > d > 2 - a_1$  multiplied by the probability of this event. If the other firm's price is greater than firm 1's highest price then firm 1 simply sells all units at the bid prices. If the price is less than firm 1's highest price then firm 1 sells all units priced below  $p_1$  at the bid prices (as  $d > 2 - a_1$ ) and some of the units priced at  $p_1$ . The expected quantity he sells at  $p_1$  is  $d^1 - (2 - a_1)$ . The second line is the expected profits conditional on  $2 - a_1 > d > 2 - a_1 - a_2$  multiplied by the probability of this event. As before if the other firm's price is greater than firm 1's highest price then firm 1 simply sells all units at the bid prices. If the other firm's price is between  $p_1$  and  $p_2$  then firm 1 sells all units priced below  $p_1$  at bid prices and none of the units priced at  $p_1$ , as  $d < 2 - a_1$  and firm 2 will therefore supply the remainder of the units. If firm 2's price is less than  $p_2$  then

firm 1 sells all units priced below  $p_2$  at the bid prices (as  $d > 2 - a_1 - a_2$ ) and some of the units priced at  $p_2$ . The expected quantity he sells at  $p_2$  is  $d^2 - (2 - a_1 - a_2)$ . The third line gives the general term for the expected profits conditional on  $2 - \sum_{j=0}^{l-1} a_j > d > 2 - \sum_{j=0}^l a_j$  multiplied by the probability of this event.

Now consider the terms in the profit function that include  $p_1$ .

$$\begin{aligned}
& pr^1 \left( \int_{p_1}^{p^u} \sum_{i=1}^n a_i (p_i - c) f(p) dp \right. \\
& \quad \left. + \int_{p_v}^{p_1} \sum_{i=2}^n a_i (p_i - c) + (d^1 - (2 - a_1))(p_1 - c) f(p) dp \right) \\
& + pr^2 \left( \int_{p_1}^{p^u} \sum_{i=1}^n a_i (p_i - c) f(p) dp + \int_{p_2}^{p_1} \sum_{i=2}^n a_i (p_i - c) f(p) dp \right) \\
& \quad \vdots \\
& + pr^l \left( \int_{p_1}^{p^u} \sum_{i=1}^n a_i (p_i - c) f(p) dp + \int_{p_2}^{p_1} \sum_{i=2}^n a_i (p_i - c) f(p) dp \right) \\
& \quad \vdots \\
& + pr^n \left( \int_{p_1}^{p^u} \sum_{i=1}^n a_i (p_i - c) f(p) dp + \int_{p_2}^{p_1} \sum_{i=2}^n a_i (p_i - c) f(p) dp \right).
\end{aligned}$$

But  $pr^1 + pr^2 + \dots + pr^n = 1$ . The expression then simplifies to,

$$\begin{aligned}
& (1 - F(p_1))(a_1(p_1 - c) + a_2(p_2 - c) + \dots + a_n(p_n - c)) \\
& + F(p_1)(a_2(p_2 - c) + \dots + a_n(p_n - c)) \\
& + pr^1 F(p_1)(d^1 - (2 - a_1))(p_1 - c) \\
& = (1 - F(p_1))a_1(p_1 - c) + pr^1 F(p_1)(d^1 - (2 - a_1))(p_1 - c).
\end{aligned}$$

Substituting for  $F(p)$  using (6) gives,

$$\left( -\frac{E(d) - 1}{2 - E(d)} a_1 + pr^1 (d^1 - (2 - a_1)) \frac{1}{2 - E(d)} \right) p_1.$$

Let  $d^1$  be  $E(d \mid 2 - a_1 > d)$ . Then  $E(d) = (1 - p^1) d^{-1} + p^1 d^1$ . Substituting in for  $E(d)$  in the numerator and simplifying gives,

$$\frac{(-p^1(2 - d^1)(1 - a_1) - (1 - p^1)(d^{-1} - 1)a_1)p_1}{2 - E(d)}.$$

Hence

$$\frac{\partial p(P)}{\partial p_1} = \frac{-p^1(2 - d^1)(1 - a_1) - (1 - p^1)(d^{-1} - 1)a_1}{2 - E(d)} < 0. \quad (7)$$



Firm 1 can therefore increase profits by reducing the highest price towards the second highest price. If he sets  $p_1=p_2$  then we have a new price vector with  $n-1$  prices. Firm 1 can then gain by reducing the new highest price towards the second highest price. Repeating the argument  $n-1$  times, firm 1 maximises profits by reducing all bids to  $p_n$ . Hence, given firm 2 is using the mixed strategy, firm 1 will optimise by submitting a single price between  $p_v$  and  $p_u$  for all units. From the symmetry of the game the same applies for firm 2 if firm 1 is using the mixed strategy and we therefore have a mixed-strategy equilibrium. QED.

**Proof of proposition 8:** Suppose firm 2 is submitting a price  $p \in (p_v, p^u)$  for all units, according to the distribution function  $G(p)$ . Let  $g(p)$  be the corresponding density function. Then player 1's expected payoff from submitting a price  $p_1$  for all units is

$$\begin{aligned} \Pi(p_1) = & p^+ \left( \int_{p_1}^{p^u} (p_1 - c) g(p) dp + \int_{p_v}^{p_1} (p_1 - c) (d^+ - 1) g(p) dp \right) \\ & + p^- \int_{p_1}^{p^u} (p_1 - c) d^- g(p) dp \end{aligned}$$

In equilibrium  $\pi'(p)=0$  for all  $p \in (p_v, p^u)$ . This gives

$$(p - c)g(p) + G(p) = \frac{p^+ + p^- d^-}{p^+ (2 - d^+) + p^- d^-}.$$

The unique solution of this differential equation with boundary condition  $G(p^u)=1$  is

$$\begin{aligned} g(p) &= \frac{(d^+ - 1)p^+}{(p^+ (2 - d^+) + p^- d^-)} \frac{(p^u - c)}{(p - c)^2}, \\ G(p) &= \frac{p^+ + p^- d^-}{(p^+ (2 - d^+) + p^- d^-)(p - c)} \\ &\quad - \frac{(d^+ - 1)p^+ p^u + (p^+ (2 - d^+) + p^- d^-)c}{(p^+ (2 - d^+) + p^- d^-)(p - c)}. \end{aligned} \tag{8}$$

Solving  $G(p_v)=0$  gives  $p_v = (p^+(d^+-1)p^u + (p^+(2-d^+)+p^-d^-)c)/(p^++p^-d^-)$ . For simplicity we only consider whether firm 1 can gain by submitting two prices,  $\{p_1, p_2\}$  where  $p_1 > p_2$ . Let  $a$  be the quantity

bid at  $p_1$ . Then firm 1's profit given firm 2 is using the mixed strategy is

$$\begin{aligned}
p(p_1, p_2) = & p^a \left( \int_{p_1}^{p^u} a(p_1 - c) + (1-a)(p_2 - c)g(p)dp \right. \\
& + \int_{p_V}^{p_1} (1-a)(p_2 - c) + (d^a - (2-a))(p_1 - c)g(p)dp \\
& + p^b \left( \int_{p_1}^{p^u} a(p_1 - c) + (1-a)(p_2 - c)g(p)dp \right. \\
& + \int_{p_2}^{p_1} (1-a)(p_2 - c)g(p)dp + \int_{p_V}^{p_2} (d^b - 1)(p_2 - c)g(p)dp \\
& + p^c \left( \int_{p_1}^{p^u} (d^c - (1-a))(p_1 - c) + (1-a)(p_2 - c)g(p)dp \right. \\
& + \int_{p_2}^{p_1} (1-a)(p_2 - c)g(p)dp \\
& \left. + p^d \int_{p_2}^{p^u} d^d(p_2 - c)g(p)dp \right)
\end{aligned}$$

where  $p^a = \Pr(2 < d < 2 - \alpha)$ ,  $p^b = \Pr(2 - \alpha < d < 1)$ ,  $p^c = \Pr(1 < d < 1 - \alpha)$ ,  $p^d = \Pr(1 - \alpha < d < 0)$ ,  $d^a = E(d/2 < d < 2 - \alpha)$ ,  $d^b = E(d/2 - \alpha < d < 1)$ ,  $d^c = E(d/1 < d < 1 - \alpha)$ ,  $d^d = E(d/1 - \alpha < d < 0)$ .

Taking out the terms that involve  $p_1$ , substituting for  $G(p_1)$  using (8) and simplifying as before, the first derivative of the profit function with respect to  $p_1$  is,

$$\begin{aligned}
\frac{\partial p(p_1, p_2, a)}{\partial p_1} = & -(p^a + p^b)(p^a(2 - d^a)(1 - a) + p^b(d^b - 1)a) \\
& - (d^c - (1 - a))p^c(p^a(d^a - 1) \\
& + p^b(d^b - 1)) + (d^a - (2 - a))p^a(p^c d^c + p^d d^d) / \\
& (2(p^+(d^+ - 1) + p^-(d^-))).
\end{aligned} \tag{9}$$

The case where demand is always greater than 1 is the special case where  $p^a + p^b = 1$ . The derivative is then given by (7). In that case, the derivative is always negative regardless of the demand distribution. The denominator of (9) is always positive. All three terms in the numerator of (9) are positive and the overall sign is therefore ambiguous and depends on the demand distribution. The mixed-strategy equilibrium holds for any distribution where the derivative is negative for all  $a \in [0, 1]$ .

If, however, the derivative is positive for some  $a \in [0,1]$ , then each firm can gain by submitting two prices when the other uses the mixed strategy. We now show that the mixed-strategy equilibrium holds when demand is distributed uniformly.

Assume  $d \sim U[0,2]$ . Then  $p^a = \alpha/2$ ,  $p^b = (1-\alpha)/2$ ,  $p^c = \alpha/2$ ,  $p^d = (1-\alpha)/2$ ,  $d^a = (4-\alpha)/2$ ,  $d^b = (3-\alpha)/2$ ,  $d^c = (1-\alpha)/2$ ,  $d^d = (1-\alpha)/2$ . Substituting these values into the numerator of (9) gives,

$$-\frac{1}{8}(a^2(1-a) + a(1-a^2)) < 0.$$

This is less than zero for any value of  $\alpha$  and each firm can increase profits by reducing the higher price to the lower one. We now give an example of a distribution where the equilibrium does not hold. Assume  $\alpha = .5$ ,  $d^a = 1.75$ ,  $d^b = 1.25$ ,  $d^c = .75$ ,  $d^d = .25$ . Substituting these values into the numerator of (9) gives,

$$p^a(p^d - 4p^b - 2p^a) - 2p^{b^2} - p^c p^b. \quad (10)$$

This expression is greater than zero when,

$$p^d > \frac{2p^{b^2} + p^c p^b}{p^a} + 4p^b + 2p^a. \quad (11)$$

For example, if  $p^a = .05$ ,  $p^b = .05$ ,  $p^c = .05$  and  $p^d = .85$  then the inequality holds. Each firm can then increase profits by increasing the distance between the prices. The mixed-strategy equilibrium, where they set one price for all units, no longer holds. If, however, we substitute  $p^a = .1$ ,  $p^b = .1$ ,  $p^c = .1$  and  $p^d = .7$  into (10) then the term is negative and the equilibrium continues to hold. It is clear from (9) and (11) that, for the equilibrium not to hold, the demand distribution must be heavily skewed towards low levels of demand. In particular, the value of  $p^d$  needs to be very high. For any distribution that is not skewed in this way the equilibrium holds. QED.

**Proof of proposition 12:** Assume firm 2 is bidding at  $m + c_b$ . If  $y_1^* + y_2^* > \underline{d}$  then the reaction functions (5) cross along a continuum of points where  $y_1 + y_2 = \underline{d}$ . Substituting for  $y_1^*$  and  $y_2^*$  from (4) and solving for  $\underline{d}$  yields the inequality. If it is an equilibrium to set the total capacity at  $\underline{d}$  when an

increase in quantity yields a chance that the marginal price will be reduced to  $m+c_b$ , then it is also an equilibrium if the base-load price bids are less than  $m+c_b$ . QED.

**Proof of proposition 13:** If  $y_1^*+y_2^* > \underline{d}$ , then the reaction functions are given by (3). They cross at  $(y_1^*, y_2^*)$ . If  $y_1^* < \underline{d}$  then the price equilibria given by proposition 12 hold (where firm 1 is the low price

firm). Substituting for  $y_1^*$  using (4) gives,  $y_1^* < \underline{d} \Rightarrow \frac{\Delta_d(\frac{1}{2}\Delta_c + m)}{\Delta_c} < \underline{d}$ . QED.

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